TOPOLOGY II BACK PAPER EXAM

Time : 3 hours Max. Marks : 50

Answer all questions. You may use results proved in class after correctly stating them. Any other claim must be accompanied by a proof.

(1) Decide whether the following statements are *true* or *false*. Answers without correct and complete justifications will not be awarded any marks.

(a) If G is a finitely generated group, then G has only finitely many subgroups of a given finite index. (essentially topological proofs only).

(b) If $p : \mathbb{R}P^2 \times \mathbb{R}P^4 \longrightarrow X$ is a covering, then X is homeomorphic to $\mathbb{R}P^2 \times \mathbb{R}P^4$.

(c) If e is an open cell in a CW-complex X and f a characteristic map of e, then $\dot{e}$ is a deformation retract of $\bar{e} - f(0)$.

(d) The space

$$X = (\{0,1\} \times [0,1]) \cup (\{1/n\}_{n \geq 1} \times [0,1]) \cup ([0,1] \times \{0,1\})$$

with the subspace topology of $\mathbb{R}^2$ is a CW-complex.

(e) If $f : S^1 \times D^2 \longrightarrow S^3$ is an embedding, then $H_1(S^3 - f(S^1 \times D^2); \mathbb{Z}) \cong \mathbb{Z}$. [5x4=20]

(2) If X is a finite CW-complex prove that the Euler charateristic $\chi(X)$ is also given by

$$\chi(X) = \sum_{i \geq 0} (-1)^i \alpha_i$$

where α_i is the number of i-cells in X. [10]

(3) Compute the homology groups of $\mathbb{R}P^n$. A detailed proof is expected. [10]

(4) Let $n > k \geq 0$. Let $f : S^k \longrightarrow S^n$ be an embedding. Compute the homology groups

$$\widetilde{H}_i(S^n - f(S^k); \mathbb{Z}).$$

Using this, state and prove the generalized Jordan curve theorem. You may assume the fact that a k-cell in S^n does not separate S^n. [5+5]